

Let  $c \geq 0$  and define the  $n \times n$  matrix

$$A_c = \begin{pmatrix} 1+c & c & \cdots & c \\ c & 1+c & \cdots & c \\ \vdots & & \ddots & \vdots \\ c & c & \cdots & 1+c \end{pmatrix}.$$

The purpose of this problem is to analyze the convergence of the classical iterations for  $A_c x = b$ . Justify your results (you may use results proved in class).

- a) Show that  $A_c$  is SPD for all values of  $c \geq 0$ .
- b) Determine for which values of the parameter  $\omega$  Richardson iteration converges for  $A_c$ .
- c) Is the Jacobi iteration convergent for  $A_c$  for all  $c$ ? If not, for which  $c$  is it convergent?
- d) Is the Gauss-Seidel iteration convergent for  $A_c$ ? If not, for which  $c$  is it convergent?

- e) As a concrete example, what are the results of b)–d) for the matrix  $\begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{pmatrix}$ ?